

Position Space Versions of Magueijo-Smolín Doubly Special Relativity Proposal and the Problem of Total Momentum.

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We present and discuss two different possibilities to construct position space version for Magueijo-Smolín (MS) doubly special relativity proposal. The first possibility is to start from ordinary special relativity and then to define conserved momentum in special way. It generates MS invariant as well as nonlinear MS transformations on the momentum space, leading to consistent picture for one-particle sector of the theory. The second possibility is based on the following observation. Besides the nonlinear MS transformations, the MS energy-momentum relation is invariant also under some inhomogeneous linear transformations. The latter are induced starting from linearly realized Lorentz group in five-dimensional position space. Particle dynamics and kinematics are formulated starting from the corresponding five-dimensional interval. There is no problem of total momentum in the theory. The formulation admits two observer independent scales, the speed of light, c , and k with dimension of velocity. We speculate on different possibilities to relate k with fundamental constants. In particular, expression of k in terms of vacuum energy suggests emergence of (minimum) quantum of mass.

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I. INTRODUCTION

It has been discussed in a number of papers (see [1-3] and references therein) that some experimental data at higher energies may be considered as pointing on deviations from special relativity kinematics. To describe the deviations, one can add rotationally invariant (but not Lorentz invariant) interaction terms with some tiny parameters to the standard model Lagrangian. It leads, in particular, to deformation of the special relativity energy-momentum relation [1]. There are at least two possibilities to treat the situation.

A. The noninvariant terms indicates on existence of preferred frame (the frame in which the cosmic microwave background is isotropic, so the interaction is rotationally invariant [1]), thus the Lorentz symmetry is violated at higher energies.

B. Other point of view, which implies preservation of the principle of relativity of inertial frames, has been put forward in the works [3-5]. The underlying symmetry group remains, as before, the Lorentz group, but it's realization on the momentum space is supposed to be different from the ordinary one. The deformed realization involves tiny parameter in such a way, that one recovers the standard Lorentz transformations in some limit. The parameter turns out to be one more observer independent scale of the theory in addition to the speed of light,

from which it follows the name "doubly (deformed) special relativity" (DSR). In particular, the scale introduced in [4, 5] was identified with the Planck energy.

There is a number of attractive motivations for such a kind modification, some of them are enumerated below.

1. Mathematical motivation. To relate the ordinary Lorentz group realization

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}, \quad (1)$$

with transformations among inertial observers (namely with the boosts), one introduces the parameter c : $x^0 = ct$, which has rather special properties in the resulting theory: a) the transformations degenerate in the limit $V \rightarrow c$; b) the limit $c \rightarrow \infty$ reproduce the Galileo relativity; c) the velocity $v = c$ turns out to be observer independent scale of the theory (namely, maximum signal velocity). Then one asks on generalization of the special relativity which admits more than one dimensional parameter with similar properties.

2. Physical motivation. Several quantum gravity calculations indicate on modification of the special relativity energy-momentum relation (suppressed by the Planck scale). Then one can believe on DSR as an intermediate theory where the quantum gravity effects are presented even in the regime of negligible gravitational field [5, 6].

3. "Experimental" motivation. The modified energy-momentum relation implies corrections to the GZK cut-off (or even absence of the latter [1]), so DSR proposals may be relevant for discussion of the threshold anomalies in ultra high-energy cosmic rays [3, 5]. Possible energy dependence of speed of light in DSR theory may be relevant to planned gamma-ray observations [5]. Some other astrophysics data were discussed in the DSR framework, see [7, 3, 2].

The DSR proposals [3-5] have been formulated in space of conserved energy-momentum p^{μ} , that is as a list of kinematical rules of the new theory. In particular,

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Magueijo-Smolin (MS) suggestion [4, 5] is to take the momentum space realization of the Lorentz group in the form

$$\Lambda_U = U^{-1} \Lambda U, \quad p'^{\mu} = (\Lambda_U)^{\mu}_{\nu} p^{\nu}, \quad (2)$$

where Λ represents ordinary Lorentz transformation and $U(p^{\mu}, \lambda)$ is some operator which depends on the invariant scale λ . Ordinary energy-momentum relation $(p^{\mu})^2 = -m^2$ is not invariant under the realization and is replaced by $[U(p^{\mu})]^2 = -m^2$. It suggests kinematical predictions different from that of special relativity.

Unfortunately, the list of kinematical rules of MS is not complete. Central problem of the DSR kinematics is consistent definition of total momentum for many particle system (see [5, 8]). Actually, due to non-linear form of the transformations, ordinary sum of momenta does not transform as the constituents. Different covariant composition rules proposed in the literature lead to hardly acceptable features [8], in particular, one is faced with the "soccer ball problem". One possibility to avoid the problem was proposed in the recent work [12].

To discuss physical interpretation of the DSR kinematics it is desirable to find the underlying space-time version of the theory, that is to construct realization of the Lorentz group in position space, and then to formulate dynamical problems which generate the DSR kinematics in one or another way. The issue turns out to be rather delicate question, as it was discussed in [9-12]. In this work we discuss two different possibilities to construct the space-time version of the MS DSR kinematics. To find it one needs, in fact, to decide what is the relation between x^{μ} and the conserved energy-momentum p^{μ} . The relation is absent in the MS construction and can be different now from the standard one. In Section 2 we suppose that the underlying version is ordinary special relativity and then deform definition of the momentum in an appropriate way. It generates the MS invariant as well as the nonlinear MS transformations on the momentum space, leading to consistent picture for one-particle sector of the theory. In Section 3 we consider the opposite possibility: one takes the standard definition of the momentum and then deforms the Lorentz group realization. We present linear realization of the Lorentz group in five-dimensional position space which leads to the MS energy-momentum relation. Particle dynamics and kinematics are formulated starting from the corresponding five-dimensional interval. We point that there is no problem of total momentum in the theory. The formulation admits two observer independent scales, the speed of light, c , and k with dimension of velocity. In Section 4 we discuss different possibilities to identify k with fundamental constants.

II. MS DOUBLY SPECIAL RELATIVITY STARTING FROM THE SPECIAL RELATIVITY

Imposing that the special relativity can be deformed by the scale λ , one can write for the conserved momentum the expression $p^{\mu} = m f^{\mu}_{\nu}(x^{\mu}, \dot{x}^{\mu}, \lambda) \dot{x}^{\nu}$ with $f^{\mu}_{\nu} \xrightarrow{\lambda \rightarrow 0} \delta^{\mu}_{\nu}$. Assuming this, one can try to reproduce the given kinematics starting, for instance, from ordinary special relativity in the position space. The resulting theory is discussed in this Section. It will be demonstrated that MS one particle kinematics can actually be considered as corresponding to the ordinary special relativity particle dynamics, if one deforms definition of the special relativity conserved momentum in an appropriate way. The MS invariant and the corresponding nonlinear transformations are induced from the standard ones for x^{μ} variables.

A. Initial MS proposal

The initial MS proposal [4] implies that all the inertial observers should agree to take the deformed dispersion relation for the conserved momentum of a particle

$$p^2 = -m^2 c^2 (1 + \lambda p^0)^2, \quad (3)$$

where λ is some observer independent scale. Then one needs to find realization of the Lorentz group on the momentum space which leaves Eq.(3) covariant. They have suggested the following nonlinear transformations:

$$p'^{\mu} = \frac{\Lambda^{\mu}_{\nu} p^{\nu}}{1 + \lambda(p^0 - \Lambda^0_{\nu} p^{\nu})}. \quad (4)$$

Now, one notes that $p^{\mu} = m(1 + \lambda p^0) \dot{x}^{\mu}$, being substituted in Eq.(3), gives the standard relation $(\dot{x}^{\mu})^2 = -c^2$. Solution of the equation is

$$p^{\mu} = \frac{m \dot{x}^{\mu}}{1 - \lambda m \dot{x}^0}. \quad (5)$$

The standard Lorentz transformation (1) for x generates the transformation (4) for p defined in accordance with Eq.(5).

Equations of motion for x^{μ} can be restored from Eq.(3) and the condition of momentum conservation $\dot{p}^{\mu} = 0$

$$S^{\mu} \equiv \left(\frac{\dot{x}^{\mu}}{1 - \lambda m \dot{x}^0} \right) = 0, \quad \dot{x}^{\mu} \dot{x}_{\mu} = -c^2. \quad (6)$$

It gives space-time version of (3), (4) with the conserved momentum defined by Eq.(5). Let us point that the first equation in (6) is covariant with respect to (1): $S^{\mu}(x') = B^{\mu}_{\nu} S^{\nu}(x)$, with B -matrix being invertible on the same region as (5). The system (6) describes the free particle motion, which can be confirmed by direct computation (in fact, the first equation in (6) can be substituted by $\ddot{x}^{\mu} = 0$).

B. General MS proposal

In general case (2) one takes some particular form of the operator $U(p, \lambda)$ to obtain the deformed energy-momentum relation and the corresponding Lorentz group realization in the momentum space. It is not difficult to generate these models by using of the trick described in the previous subsection, that is starting from the special relativity particle dynamics. Let us consider first the polynomial energy-momentum relation

$$p^\mu p_\mu = -m^2 c^2 \left(1 + \sum_{n=1}^N \alpha_n (\lambda p^0)^n\right)^2. \quad (7)$$

One writes the equations for determining the momenta in terms of \dot{x}^μ

$$\begin{aligned} p^0 &= m \left(1 + \sum_{n=1}^N \alpha_n (\lambda p^0)^n\right) \dot{x}^0, \\ p^i &= m \left(1 + \sum_{n=1}^N \alpha_n (\lambda p^0)^n\right) \dot{x}^i. \end{aligned} \quad (8)$$

Assuming that the first equation can be resolved in relation of p^0 , one finds solution of the system (8) in the form

$$p^0 = p^0(\dot{x}^0, \lambda), \quad p^i = p^i(\dot{x}^i, \dot{x}^0, \lambda). \quad (9)$$

Then the standard Lorentz transformations for x^μ (1) generate some nonlinear realization of the Lorentz group on the momentum space (9). By construction, the dispersion relation (7) is invariant with respect to this realization.

In general case the energy-momentum relation is of the form [5]

$$(p^0)^2 f_1^2(p^0, \lambda) - (p^i)^2 f_2^2(p^0, \lambda) = m^2 c^2. \quad (10)$$

Equations for determining the momenta acquire the form $p^0 f_1(p^0, \lambda) = m \dot{x}^0$, $p^i f_2(p^0, \lambda) = m \dot{x}^i$. Assuming that the first equation can be resolved in relation of p^0 , one finds solution of the system in the form (9).

Thus we have demonstrated that MS construction can be considered as corresponding to space-time particle dynamics of the special relativity, that is the kinematics is generated starting from $\dot{x}^2 = -c^2$, $\ddot{x}^\mu = 0$, $\dot{x}'^\mu = \Lambda^\mu{}_\nu \dot{x}^\nu$, which gives a consistent picture in one-particle sector. In ordinary special relativity, expression for the conserved momentum is dictated by the translation invariance. Then, how one can take some different expression of the type (5)? The point is that the MS kinematics is well defined for the one particle sector only. In this case definition of conserved momentum is conventional (since $\dot{x}^\mu = \text{const}$, any function of \dot{x}^μ can be taken as the conserved quantity). For many-particle case, definition of the conserved momentum is not conventional. On absence of consistent addition rule in many-particle sector of the MS kinematics, the presented point of view,

being quite simple, seems to be unreasonable. On this reasoning we propose below other position version, which generates the MS invariant on momentum space. In particular, the version turns out to be free of the problem of total momentum.

III. DOUBLY SPECIAL RELATIVITY AS $M^4 \times \mathbb{R}$ "GALILEAN" RELATIVITY

Discussion of this Section is based on the following two observations.

1. According to the previous analysis, the problem of total momentum is due to the fact that the conserved momentum, being defined in MS DSR in terms of the velocities in the nonlinear form (5), is not a tangent vector. To avoid the problem, one preserves the ordinary definition of the momentum, deforming the space-time interval in such a way that the resulting theory reproduce the MS invariant (3) in the momentum space. Analysis of the corresponding particle dynamics reveals that the evolution parameter T of the model does not coincide with proper time of the particle, see Subsection 3.1. So, in this framework, the deformed dispersion relation suggests emergence of one more evolution parameter in the position version of DSR theory. One has x^0 , τ , T , instead of x^0 , τ of the special relativity theory. It prompts that the appropriate group-theoretic framework for the model may be some realization of the Lorentz group in 5-dimensional position space.

2. Besides the nonlinear symmetry (4), the MS relation (3) is invariant also under the linear inhomogeneous transformations

$$\begin{aligned} p'^0 &= \Lambda^0{}_0 p^0 + \frac{\Lambda^0{}_i p^i}{\sqrt{1 - c^2 \lambda^2}} - (1 - \Lambda^0{}_0) \frac{c^2 \lambda}{1 - c^2 \lambda^2} \\ p'^i &= \sqrt{1 - c^2 \lambda^2} \Lambda^i{}_0 p^0 + \Lambda^i{}_j p^j + \Lambda^i{}_0 \frac{c^2 \lambda}{1 - c^2 \lambda^2}. \end{aligned} \quad (11)$$

It suggests linear realization of the Lorentz group in the position space. We consider below the position version which leads to momentum transformations with more symmetric structure, see Eqs.(14), (23). In contrast to the MS case, the resulting model has no degeneration in the limit of a particle with large mass. Generalization on the MS case is straightforward and will be presented in a separate work.

A. Particle dynamics based on the MS invariant

Assuming the standard relation among momenta and velocities $p^\mu = m \frac{dx^\mu}{dT}$, the equations of motion

$$\frac{d^2 x^\mu}{dT^2} \equiv \ddot{x}^\mu = 0, \quad \dot{x}^\mu \dot{x}_\mu = -c^2 (1 - \lambda m c \dot{x}^0)^2, \quad (12)$$

corresponds to the MS kinematics. Here T is invariant evolution parameter. Analysis of the dynamics is similar

to that of ordinary relativistic particle [13]. One takes the deformed relation $dx^0 = c(1 - \lambda^2 m^2 c^4)^{-\frac{1}{2}} dt$, otherwise the invariant velocity scale depends on the rest mass. The relation degenerates for $m^2 \rightarrow \frac{1}{\lambda^2 c^4}$, which represents the position version of the "soccer ball problem" (the model considered below is free of this problem). Solution of Eq.(12) is $x^i(t) = v^i t + a^i$, where the three-velocity is restricted by $|v^i| \leq c$. From the second equation in (12) it follows, that the maximum velocity c turns out to be the invariant scale: if a particle has $|v^i| = c$ for one observer, it has the same velocity for any other observer. The same equation allows one to relate proper time of the particle dt with the evolution parameter

$$dT = \sqrt{\frac{1 + \lambda m c^2}{1 - \lambda m c^2}} dt. \quad (13)$$

As it was mentioned above, the proper time does not coincide with the evolution parameter, similarly to geodesic particle motion in gravity theory.

B. Realization of the Lorentz group on $M^4 \times \mathbb{R}$ -space

Let us consider a realization of the Lorentz group $\{\Lambda^\mu{}_\nu\}$ in 5-dimensional position space $M^4 \times \mathbb{R}$ (parameterized by (x^μ, x^5) , $\eta_{\mu\nu} = (-+++)$): $x'^A = T^A{}_B(\Lambda)x^B$, namely

$$\begin{aligned} x'^\mu &= \Lambda^\mu{}_\nu x^\nu + (\delta^\mu{}_0 - \Lambda^\mu{}_0)x^5, \\ x'^5 &= x^5. \end{aligned} \quad (14)$$

It leaves x^5 invariant. Let us enumerate some properties of the DSR theory based on Eq.(14).

1- Let us note that deformations of the special relativity in some domain by means of the transformation $\Lambda_{def} = U^{-1}\Lambda U$ suggest existence of (singular) change of variables $x_{SR} = U^{-1}x$. The variable x_{SR} has the standard transformation law under Λ_{def} : $x_{SR} = \Lambda x_{SR}$. It is true for the Fock-Lorentz realization [14] and for the recent DSR proposals [9, 12]. Moreover, different DSR proposals in momentum space can be considered either as different definitions of the conserved momentum p^μ in terms of the de Sitter momentum space variables η^A [8, 11], or as different definitions of p^μ in terms of the special relativity velocities $v^\mu = \frac{dx^\mu}{d\tau}$, see Section 2. Thus the known DSR proposals state, in fact, that experimentally measurable coordinates can be different from the ones specified as "measurable" by the special relativity theory. For the case under consideration, Eq.(14) suggests, in fact, redefinition of the special relativity observer time $x_{SR}^0 = x^0 - x^5$.

2- Eq.(14) is realization of the Lorentz group: $T(\Lambda_1)T(\Lambda_2) = T(\Lambda_1\Lambda_2)$, with $\det T \neq 0$.

3- The transformations with $\Lambda^\mu{}_\nu = (\Lambda^0{}_0 = 1, \Lambda^0{}_i = \Lambda^i{}_0 = 0, \Lambda^i{}_j \equiv R^i{}_j, R^T = R^{-1})$ are identified with space rotations of M^4 : $x'^i = R^i{}_j x^j$, $x'^0 = x^0$, $T' = T$. They are not deformed by means of x^5 .

4- One notes similarity of Eq.(14) with the Galileo transformations in $\mathbb{R}^3 \times \mathbb{R}$: $x'^i = G^i{}_j x^j - V^i t$, $t = t'$, but in (14) it was taken the Minkowski space M^4 instead of \mathbb{R}^3 .

5- To make the physical interpretation of Eq.(14), one assumes that it describes the transformation law among inertial observers. To construct the covariant dynamics, all the observers should agree on taking x^5 as the evolution parameter, the latter is independent on a frame in accordance with Eq.(14). Thus one takes $x^5 = kT$, where $[T] = \text{sec}$ and k is some constant $[k] = \frac{m}{\text{sec}}$ (the notations $\dot{x} = \frac{dx}{dT}$ will be used below). The special relativity then corresponds to the limit $k \rightarrow 0$ in Eq.(14). Note that the choice $x^5 = cT$ is not interesting in this respect. The additional dimension x^5 decouples from M^4 in the limit.

6- The invariant interval of the transformations (14) is

$$-ds_5^2 = \eta_{\mu\nu} dx^\mu dx^\nu + (dx^5 - 2dx^0)dx^5, \quad (15)$$

and leads to deformed energy-momentum relation in the momentum space, see Eq.(22) below. The MS relation corresponds to the interval $\eta_{\mu\nu} dx^\mu dx^\nu + (dx^5 + dx^0)^2$.

7- Similarly to the Magueijo-Smolín [4], let us ask on existence of the tangent space vector with zero component preserved by the modified action of the Lorentz group. From Eq.(14) and from the requirement $\dot{x}'^0 = \dot{x}^0$ one finds the unique solution $\dot{x}^\mu = (k, 0, 0, 0)$. Thus k represents some observer independent scale of the model.

To conclude this subsection, we point that the conserved momentum can be defined in the standard form

$$p^A = m\dot{x}^A = (m\dot{x}^\mu, mk), \implies p'^A = T^A{}_B(\Lambda)p^B \quad (16)$$

Then $P^A = p^A + \pi^A$ is the covariant composition rule (P obeys the Eq.(16)), and P^0 is unbounded from above, in contrast with the MS theory. The model turns to be free of the problem of total momentum.

C. Covariant dynamics of $M^4 \times \mathbb{R}$ particle

To describe the free particle motion, one takes $\frac{d^2 x^\mu}{dT^2} = \ddot{x}^\mu = 0$. Similarly to the special relativity theory, one needs to specify the relation among T and the observer time x^0 . It can be achieved by using of the invariant interval (15) which we have in our disposal. Since the interval involves one more variable, the latter must be specified also. Let us write Eq.(15) in the form

$$\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -(\dot{s}_5^2 - k^2 + 2k\dot{x}^0). \quad (17)$$

To obtain the special relativity in the limit $k \rightarrow 0$ one takes $\dot{s}_5^2 = f(c, k)$, where $f \rightarrow c^2$ when $k \rightarrow 0$. One notes that the equation for \dot{s}_5^2 is invariant with respect to the transformations (14). In particular, taking $\dot{s}_5^2 = c^2 + k^2$ one obtains the following relativistic dynamics

$$\ddot{x}^\mu = 0, \quad \eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -c^2 \left(1 + \frac{2k}{c^2} \dot{x}^0\right). \quad (18)$$

Analysis of the dynamics is similar to that of ordinary relativistic particle [13]. One notes that $\ddot{x}^0 = 0$ is consequence of other equations in (18) for $\dot{x}^0 \neq k$. The Cauchy problem for the remaining equations is $x^i(T = 0) = a^i$, $x^0(T = 0) = 0$, $\dot{x}^i(T = 0) = b^i$, for any a^i , b^i , where the clocks synchronization is implied. One finds the solution

$$x^i = b^i T + a^i, \quad x^0 = (k + \sqrt{k^2 + c^2 + (b^i)^2})T. \quad (19)$$

Assuming that these expressions describe a particle motion in parametric form, one finds, taking the standard relation $dx^0 = cdt$

$$x^i(t) = v^i t + a^i, \quad v^i = \frac{cb^i}{k + \sqrt{k^2 + c^2 + (b^i)^2}}. \quad (20)$$

where v^i represents the initial three-velocity. The latter is restricted: $|v^i| \rightarrow c$ when $|b^i| \rightarrow \infty$. From the second equation in (18) one finds, that the maximum velocity c turns out to be the invariant scale: if a particle has $|v^i| = c$ for one observer, it has the same velocity for any other observer. The same equation allows one to relate proper time of the particle with the evolution parameter

$$dT = \left[\sqrt{1 + \left(\frac{k}{c}\right)^2} - \frac{k}{c} \right] dt, \quad (21)$$

where the proportionality factor is non degenerated.

In the momentum space one finds the deformed energy-momentum relation

$$p^2 = -m^2 c^2 \left(1 + \frac{2k}{mc^2} p^0 \right), \quad (22)$$

which is invariant under the transformations generated by (15)

$$p'^\mu = \Lambda^\mu{}_\nu p^\nu + (\delta^\mu{}_0 - \Lambda^\mu{}_0)mk. \quad (23)$$

IV. DISCUSSION: IDENTIFYING k WITH FUNDAMENTAL CONSTANTS

In this work we have proposed and discussed two different position space versions for the MS DSR kinematics. One possibility is to start from ordinary special relativity and then to define the conserved energy and momentum in special way (see Eq.(5) for the initial MS proposal). It generates the MS invariant (3) as well as the MS transformations (4) on the momentum space, leading to consistent picture for one-particle sector of the theory. Generalization on multi-particle sector is problematic, mainly due to the fact that it is not yet known a consistent rule for addition of momenta [8].

The problem of total momentum can be avoided, if one preserves ordinary definition for the conserved momentum. It implies deformation of the special relativity interval as well as the Lorentz group realization in the

position space. Following this line, we have presented linear realization of the Lorentz group in five-dimensional position space (14), with the fifth coordinate being invariant under the transformations. Particle dynamics and kinematics were formulated starting from the five-dimensional interval. In particular, the model leads to the MS-type energy-momentum relation (22). This proposal can be compared with interpretation of DSR kinematics in terms of pentamomentum of (Anti)de Sitter space [16]. Due to linear realization of the Lorentz group on the space, the sum rule for composite system is simply add the pentamomentum of the constituents. It leads to rescaling of k for composite systems and reproduces the sum rule for quadrimomentum proposed in [5]. This interpretation implies also a concept of five-dimensional spacetime, with the fifth coordinate being characteristic of a reference frame mass [16].

The position version constructed in Section 3 naturally leads to two observer (and position) independent scales with dimension of velocity: c and k . The scale k is supposed to be small, and to obtain the special relativity in the limit $k \rightarrow 0$ one identifies c with speed of light. Now one asks on interpretation of k in terms of other fundamental constants. There are three dimensional constants which should play a fundamental role in the quantum theory of gravity: speed of light c , Newton's gravitational constant G , and the Planck constant \hbar . They can be used to construct the Planck scales, of length $l_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}m$ (or energy $E_p = \sqrt{\frac{\hbar c^5}{G}} \sim 10^8 \frac{kgm^2}{seg^2}$), time $t_p = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-43}seg$, and mass $M_p = \sqrt{\frac{\hbar c}{G}} \sim 10^{-8}kg$. Besides this, a possibility to include some other dimensional scales have been discussed [7, 15, 12]. In particular, in the work [7] it was analysed an algebraic construction which implies three scales c , E_p , Λ , with E_p identified with the Planck energy and Λ being the cosmological constant. The latter appears also in a natural way in position version of the DSR theory based on conformal group [12]. So, we include the cosmological constant in our subsequent analysis.

Let us discuss different possibilities to relate k with the fundamental constants.

1. On dimension grounds, one writes the scale k in terms of the Planck energy $k = \frac{mc^3}{2E_p}$. It implies the energy-momentum relation $p^2 = -m^2 c^2 (1 + \frac{cp^0}{E_p})$, as well as dependence of the Lorentz boosts (14) on rest mass

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu + (\delta^\mu{}_0 - \Lambda^\mu{}_0) \frac{mc^2}{2E_p} cT. \quad (24)$$

The value of k for the proton is of order 10^{-11} . For a body with rest mass being of order of the Planck mass, the DSR results diverge from that of special relativity. Thus, one is faced with the "soccer ball problem".

2. By using of the cosmological constant it is possible to avoid appearance of the mass in the transformation low, one takes $k = \sqrt[3]{\Lambda G \hbar}$. For the case, k is of order

10^{-32} . Deviation from the special relativity dynamics does not depend on the rest mass, see Eq.(18).

3. One more possibility is $k = \frac{E_{vac}}{mc}$, where the vacuum energy is $E_{vac} = \Lambda \sqrt{\frac{\hbar^3 G}{c}}$. One has

$$p^2 = -m^2 c^2 \left(1 + 2 \frac{E_{vac} p^0}{m^2 c^3}\right), \quad (25)$$

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + (\delta^{\mu}_0 - \Lambda^{\mu}_0) \frac{E_{vac}}{m c^2} c T. \quad (26)$$

This version of the theory approaches to the special relativity for macroscopic bodies. The transformations (26) suggest a lower bound for an energy of a point under observation, that is minimum quantum of mass.

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